Dot Product-Angle Formula

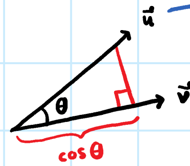
Wednesday, May 10, 2023 9:33 AM

dot product - angle formula:

if angle between $\vec{u}$ & $\vec{v}$ is θ, then ...

$$\vec{u}\cdot\vec{v} = |\vec{u}|\cdot|\vec{v}|\cdot\cos\theta \quad \leftarrow \text{all \#'s}$$

$$\left(\text{or } \frac{\vec{u}}{|\vec{u}|}\cdot\frac{\vec{v}}{|\vec{v}|} = \cos\theta\right)$$



if unit vectors

ex 3) find angle θ between $\vec{u} = \langle 2, 1, 4\rangle$ & $\vec{v} = \langle -1, 3, 2\rangle$

solution: use dot - angle formula:

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}|\cdot|\vec{v}|} = \frac{9}{|\vec{u}|\cdot\sqrt{14}} = \frac{9}{\sqrt{21}\cdot\sqrt{14}}$$

$$|\vec{u}| = \sqrt{(2^2)+(1^2)+(4^2)}$$

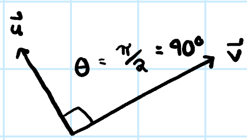
$$|\vec{u}| = \sqrt{21}$$

then $\theta = \arccos\left(\frac{9}{\sqrt{21}\cdot\sqrt{14}}\right)$

sign of $\vec{u}\cdot\vec{v}$:

- if $\vec{u}\cdot\vec{v} = 0$ then

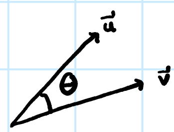
  $\vec{u}$ & $\vec{v}$ orthogonal (perpendicular) $\longrightarrow$



$$\vec{u}\cdot\vec{v} = |\vec{u}|\cdot|\vec{v}|\cos(\pi/2) = |\vec{u}|\cdot|\vec{v}|\cdot 0 = 0$$

$(\vec{u}\cdot\vec{u} = |u|^2)$ if same

- if $\vec{u}\cdot\vec{v} > 0$



less than 90°

$0 \le \theta < 90 \rightarrow \cos\theta = +$

- if $\vec{u}\cdot\vec{v} < 0$

more than 90°

$\theta > 90^\circ \rightarrow \cos\theta = -$